The final publication is available at Springer via http://dx.doi.org/10.3758/s13414-014-0830-0

Supplementary Material for Bayesian accounts of covert selective attention: a tutorial review.

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1 Formal definitions for the probabilistic generative models

Readers are referred to the Supplementary Matlab code (available from https://github. com/drbenvincent/BayesCovertAttention) to see how the models can be implemented and used in practice to generate the predictions shown in the paper.

1.1 Uncued tasks

The core part of the model is that for each trial *t*, a display type D_t is sampled from a prior distribution **p** (see the next 2 subsections). This display type then influences an observer's noisy sensory observations **x**. Noise is assumed to be normally distributed, with no bias, with target and distractor variances of σ_T^2 and σ_D^2 , respectively.

 $D_t \sim \text{Categorical}(\mathbf{p})$ (1)

$$x_{n,t} \sim \begin{cases} \text{Normal}(1, \sigma_T^2) & \text{if } n = D_t \\ \text{Normal}(0, \sigma_D^2) & \text{if } n \neq D_t \end{cases}$$
(2)

1.1.1 Uncued localisation

For localisation, the target will occur in 1 of N locations, so assuming no spatial biases the prior over display types is

$$\mathbf{p} = [\overbrace{\frac{1}{N}, \dots, \frac{1}{N}}^{N}]. \tag{3}$$

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1.1.2 Uncued yes/no task

In the yes/no task, there are N possible target present displays, and 1 target absent display. The target is present at a rate *prev*, and when it is present, the vector **s** describes the spatial prior over N stimulus locations. Therefore the prior over display types is

$$\mathbf{p} = [\overbrace{prev.s_1, \dots, prev.s_N}^N, \overbrace{1 - prev}^{P(\text{absent})}].$$
(4)

1.2 Cued tasks

Noisy sensory observations are the result of the display type as shown above in Equation 2. However, the display types are drawn from a prior distribution \mathbf{p}_t for each trial which depends upon the cue location (see next 2 subsections).

$$D_t \sim \text{Categorical}(\mathbf{p}_t)$$
 (5)

This prior distribution is a function of the cued location and the cue validity v. The cue appears uniformly at each of N spatial locations.

$$c_t \sim \text{Categorical}(\underbrace{\frac{1}{N}, \dots, \frac{1}{N}}^{N})$$
 (6)

1.2.1 Cued localisation

For the cued localisation task, the target will be in the cued location with a probability equal to the cue validity v. When the cue is invalid, the target has an equal chance of being in all non-cued locations. The prior over display types is determined by the cue location c_t

$$p_{n,t} = \begin{cases} v & \text{if } n = c_t \\ (1-v)/(N-1) & \text{if } n \neq c_t. \end{cases}$$
(7)

1.2.2 Cued yes/no

For the cued yes/no task, the prior over display types is fairly straightforward. Assuming a target prevalence rate of 50%, then regardless of the cue location, there is a 0.5 probability of a target absent $(D_t = N + 1)$ display type. The probability of a present, valid cue trial $(D_t = c_t)$ is the product of the target prevalence and cue validity, 0.5v. The probability of a target present, invalid cue trial $(D_t$ belonging to the set $(n \neq c_t) \land (n \leq N)$) is $\frac{0.5(1-v)}{N-1}$. This is summarised by the following deterministic prior over display types

$$p_{n,t} = \begin{cases} 0.5v & \text{if } n = c_t \\ \frac{0.5(1-v)}{N-1} & \text{if } (n \neq c_t) \land (n \le N) \\ 0.5 & \text{if } n = N+1. \end{cases}$$
(8)

2 Algorithmic implementation of the models

Detailed instructions are provided with the Matlab code at https://github.com/drbenvincent/ BayesCovertAttention. Two separate implementations are provided. The first is a Monte Carlo approach, similar to what has been used in previous approaches, however the novel aspect of this code is that it does not require the decision rule to be derived.

The second uses a Markov Chain Monte Carlo (MCMC) approach. While this is slower to evaluate, and the code is slightly more complex, it is much more general. Readers interested in this approach are directed to the instruction manual for the JAGS MCMC software package (Plummer, 2003), and the books by Lee and Wagenmakers (2014) and (Lunn et al., 2012).

3 The 3-stage process of calculating model predictions

Supplementary Figure 1 demonstrates the 3-step process where we use the generative graphical model to calculate predicted performance of an optimal observer (for the uncued localisation task). The process is to first generate simulated data where we know the true target locations and have some simulated internal observations \mathbf{x} (Supplementary Figure 1a). The second step is to see how well the optimal observer can then infer the display type given those noisy observations, but *without* knowledge of the true target location (Supplementary Figure 1b). The third step is to convert this inferred distribution of belief of the true location to a decision of the most likely display type \tilde{D} (Supplementary Figure 1c). The performance of the observer is then simply the proportion of trials in which the estimated display type is equal to the actual display type (state of the world) which generated the noisy observations.

References

- Lee, M. D. and Wagenmakers, E.-J. (2014). *Bayesian cognitive modeling: A practical course*. Cambridge: Cambridge University Press.
- Lunn, D., Jackson, C., Best, N., Thomas, A., and Spiegelhalter, D. J. (2012). *The BUGS book: A practical introduction to Bayesian analysis.* CRC Press.
- Plummer, M. (2003). JAGS: A program for analysis of Bayesian graphical models using Gibbs sampling. *Proceedings of the 3rd International Workshop on Distributed Statistical Computing (DSC 2003)*, pages 20–22.



Supplementary Figure 1: A 3-step procedure was used to generate predictions for all of the models in the paper, demonstrated here with the uncued localisation task. The first step is to generate a dataset of simulated target locations and corresponding noisy observations. The second step is to infer what the display type was, given the simulated sensory observations. The final step is to translate an inferred belief (the posterior distribution from step 2) to a decision. This is demonstrated for one trial as simply responding to the most probable (posterior mode). Overall this process could be called 'parameter recovery' in that we know the true display type (from step 1), but how well can an optimal observer infer this if it has to be inferred? The performance is simply the proportion of trials where the inferred display type \tilde{D} equals the true display type D.